Technical Notes

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Multiple Equilibrium Vortex Positions in Symmetric Shedding from Slender Bodies

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Introduction

THE analysis of slender axisymmetric bodies maneuvering in an airstream is usually performed by separation into axial and crossflow plane phenomena. The flow in the crossflow plane around such a body is described by flow around a two-dimensional cylinder. When the angle of attack of the body is less than 10 deg, one or more pairs of symmetric line vortices are shed on the lee side.[1,2] This is pictured, in the crossflow plane, as uniform flow around a circular cylinder, with one or more pairs of symmetrically placed, counter-rotating vortices.

The equilibrium positions of a pair of symmetric vortices shed in the wake of a circular cylinder immersed in a uniform, incompressible, and inviscid flow were first found analytically by Föppl.[3] Seath[4] developed a numerical solution for the case of N symmetric pairs of vortices behind a cylinder, and obtained a solution for equilibrium positions that was then assumed to be the only possible solution. In the present Note, it is shown that multiple equilibrium vortex configurations are possible for given cylinders and flow conditions, and specifically for the 2-symmetric-pair case, two combinations of equilibrium positions are obtained. Such multiple solutions for equilibrium positions are reminiscent of the observations of Karman vortex streets,[5] where two different equilibrium configurations are found, and this phenomenon is established theoretically[6] by means of inviscid vortex theory.

Analysis

The complex velocity, $Q=u-iv$, of the potential flowfield including the uniform flow, cylinder, and N pairs of symmetric vortices in its wake (Fig. 1) is obtained by differentiating the complex potential and is, for unit cylinder radius

$$Q=U\left(1-\frac{1}{z^2}\right)+\sum_{j=1}^{N}\frac{i\Gamma^j}{2\pi}\left[\frac{1}{z-z_j}-\frac{1}{z-(1/\bar{z}_j)}+\frac{1}{z-(1/z_j)}-\frac{1}{z-\bar{z}_j}\right] \tag{1}$$

where U is the undisturbed velocity and Γ the vortex strength. If point K is occupied by a vortex, its own induced velocity does not contribute to the linear velocity of the flowfield and has to be subtracted, giving

$$Q=\left\{U\left(1-\frac{1}{z^2}\right)+\sum_{j=1}^{N}\frac{i\Gamma^j}{2\pi}\left[\frac{1}{z-z_j}-\frac{1}{z-(1/\bar{z}_j)}-\frac{1}{z-\bar{z}_j}+\frac{1}{z-(1/z_j)}\right]-\frac{i\Gamma_K}{2\pi}\frac{1}{z-z_K}\right\}_{z=z_K} \tag{2}$$

For the vortex at K to be in equilibrium, we require that

$$Re(Q_K)=Im(Q_K)=0 \tag{3}$$

For $N=1$, Eqs. (3) can be solved analytically, resulting in the so-called[4] Föppl line, which is the locus of equilibrium positions of a single pair of vortices, when the strength is varied, as well as an additional locus of solutions on the line bisecting the cylinder perpendicularly to the flow (which we will call the "normal" line). This second locus of solutions appears in Föppl's original work but has not been mentioned in the literature. When $N=2$, Eqs. (3) are solved numerically using a "least-squares" scheme (which also retrieved the Föppl and "normal" lines for $N=1$).

Results and Discussion

Calculations based on Eq. (3) with $N=2$ were carried out for the values of Γ_1/Ua and Γ_2/Ua reported by Seath[4,7] where a is the cylinder radius. These values were chosen since the main purpose of this Note is to point out the additional vortex wakes that are possible. A systematic search for ranges of existence of such multiple vortex configurations is part of a

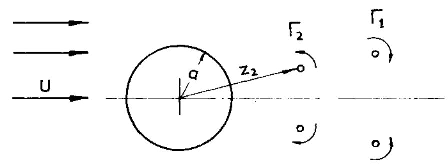

Fig. 1 Schematic description of flow in crossflow plane.

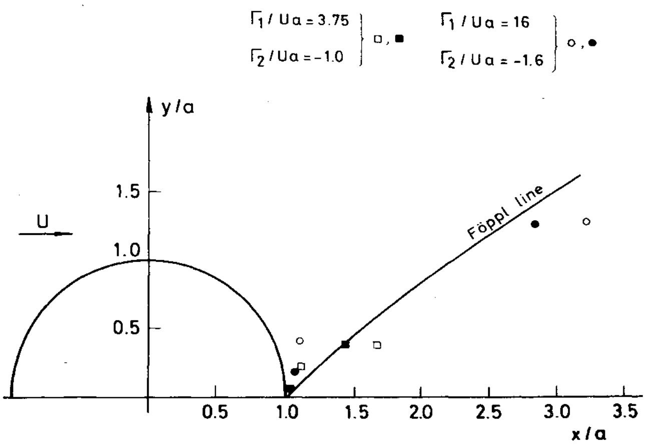

Fig. 2 Typical equilibrium configurations of two symmetric vortex pairs in the wake of a cylinder.

Received June 26, 1978; revision received Sept. 21, 1978.

Index categories: Aerodynamics; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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Table 1 Coordinates of vortices in two-pair symmetric equilibrium positions for $\Gamma_1/aU=8$, and unit cylinder radius. Subscript s stands for Seath's values, and subscript i stands for the new, alternate configurations

Γ_2/Ua	x_{1_s}	x_{1_i}	y_{1_s}	y_{1_i}	x_{2_s}	x_{2_i}	y_{2_s}	y_{2_i}
−1.7	2.101	2.073	0.693	0.693	1.162	1.157	0.205	0.194
−1.6	2.186	1.959	0.689	0.696	1.168	1.135	0.257	0.153
−1.5	2.208	1.915	0.687	0.696	1.158	1.125	0.295	0.137
−1.4	2.178	1.886	0.687	0.696	1.130	1.117	0.340	0.127
−1.3	2.090	1.864	0.688	0.695	1.081	1.110	0.375	0.118
−1.2	2.001	1.847	0.689	0.694	1.040	1.103	0.392	0.110
−1.0	1.898	1.820	0.690	0.693	0.996	1.092	0.401	0.096
−0.8	1.843	1.803	0.690	0.692	0.971	1.080	0.402	0.082
−0.3	1.783	1.778	0.690	0.689	0.935	1.047	0.398	0.047

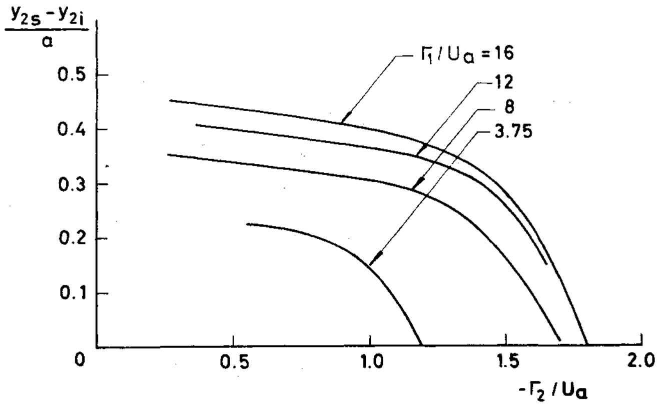

Fig. 3 Vertical distance between the two different equilibrium positions found for the inner vortices, normalized by cylinder radius vs nondimensional strength of inner vortex, with nondimensional outer vortex strength as parameter.

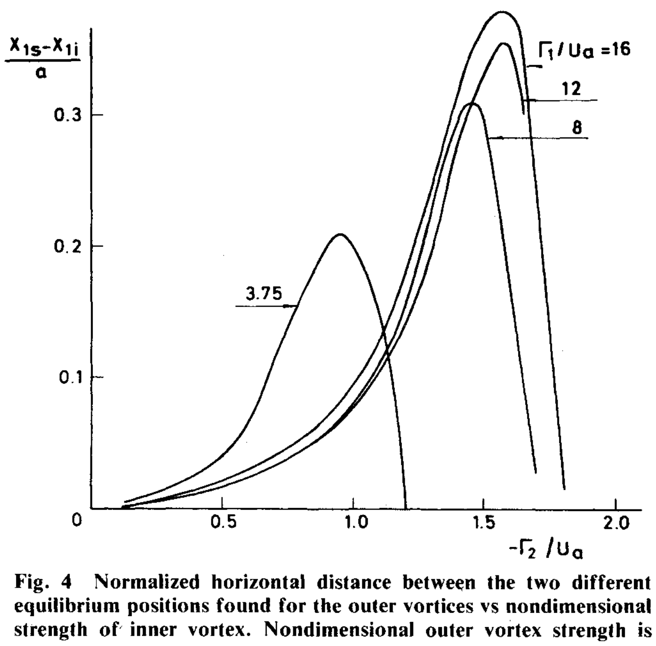

Fig. 4 Normalized horizontal distance between the two different equilibrium positions found for the outer vortices vs nondimensional strength of inner vortex. Nondimensional outer vortex strength is parameter.

more extensive investigation of stability of such vortex wakes to be reported in due course.

Two separate configurations of two equilibrium vortex pairs were found for all cases checked. These appear in part in Table 1 as well as in Fig. 2, which shows the actual equilibrium positions for one group of results. The two solutions were obtained by starting the search process from different points. Seath[7] mentions difficulties in obtaining solutions when starting from certain positions which were not found by the present method of solving the equations. However, starting points close to Föppl's line gave the Seath[7] solution, while points further away gave the additional, new solution.

Plotting the vortex positions as in Fig. 2 did not show clear tendencies in the behavior of the solutions; thus, other descriptions were sought. It was found that plotting the vertical and horizontal distances between the two separate equilibrium positions found for the same vortex gave a clear functional dependence on the nondimensional strength. Figure 3 shows the vertical distance between the equilibrium positions of the inner vortex pair. For all cases where two equilibrium positions were found, the "new" solution was closer to the axis of symmetry.

Figure 4 describes the horizontal differences in the positions of the two configurations of the outer vortex pairs, again displaying a clear pattern. Similar patterns were discerned for the other geometrical parameters, as a function of vortex strength, strengthening the conclusion that this is a systematic phenomenon and enabling prediction of these differences for other values of these parameters.

Calculations showed that both sets of solutions were stable with respect to the symmetric perturbations for some values of Γ_2, Γ_1 and unstable for others. It thus seems that both are equally prone to hydrodynamic instability, which probably results in the asymmetric wake configurations found when the angle of attack of the body is increased.

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